Comparing Different Regulatory Measures to Control Stock Market Volatility: A General Equilibrium Analysis

A. Buss
INSEAD

B. Dumas
INSEAD, CEPR, NBER

R. Uppal
Edhec, CEPR

G. Vilkov
Goethe U. Frankfurt

Banque de France

25 October 2013

Outline

Questions that We Wish to Answer

- Which policy measure is most effective for reducing excess volatility in financial markets?
  - A Tobin tax on financial transactions,
  - Shortsale constraint, or
  - Borrowing constraint.

- What is the impact of each measure on other variables?
  - **Real sector** of the economy: investment, output, consumption
  - **Bond market:** Level and term structure of interest rates
  - **Stock market**: Level of stock market, equity risk premium
  - **Trading volume** in financial markets
  - **Portfolio holdings** of individual investors.

Related Literature I

- The literature that is closest to our proposed research is the work on the remedies to the recent financial crisis:
  - Geanakoplos and Fostel (2008) and Geanakoplos (2009) study the effect of exogeneous collateral restrictions on the supply of liquidity
  - Krishnamurthy (2003) studies how credit constraints can lead to an amplification of shocks in the economy
  - Ashcraft, Gârleanu, and Pedersen (2010) compare the effectiveness of different monetary tools.
  - Alchian (1950) and Friedman (1953) on the stabilizing or destabilizing effects of speculation.

Related Literature II

- Our model is related also to the literature on "investor sentiment" and "behavioral equilibrium theory."
  - Barberis, Shleifer, and Vishny (1998) and Daniel, Hirshleifer, and Subrahmanyam (1998) have only a single group of investors who are non-Bayesian, while our model has two groups with heterogeneous beliefs who are Bayesian.
  - Hong and Stein features two investors with heterogeneous beliefs, but they are not intertemporal optimizers, in contrast to the investors in our model.

Related Literature III

- Solving general equilibrium models with incomplete markets
  - Den Haan and Marcet (1994)
  - Judd, Kubler, and Schmedders (1998)
  - Krusell and Smith (1998)
  - Special issue of the Journal of Economic Dynamics and Control (2010, vol. 1).

- Identifying equilibrium in a dynamic economy is difficult because need to solve a system of forward-backward difference equations.

- We build on method developed by Dumas and Lyasoff (2012) to show how the system of forward-backward equations can be reduced to a system of only backward (recursive) equations.

Outline

First Key Feature of Our Model: Investors with Heterogeneous Beliefs

- Hansen (2007): "While introducing heterogeneity among investors will complicate model solution, it has intriguing possibilities."
- Stiglitz (2010) criticizes representative-investor models; states importance of heterogeneous investors as key challenge.
- Sargent (2008) in his presidential address to the American Economic Association, discusses extensively the implications of the common beliefs assumption for policy.

Second Key Feature of Our Model: Heterogeneous Beliefs with Endogenous Risk

- Model meets twin challenges set by Eichenbaum (2010).
- The twin challenges Eichenbaum (2010) posed are:
  1. to model heterogeneity in beliefs and persistent disagreement between investors, and
  2. financial market frictions with risk residing internally in the financial system rather than externally in the production system.
- The twin challenges are met here because in our model the heterogeneity of investor beliefs is a fluctuating, stochastic one so that it constitutes an **internal source** of risk:
  - sentiment is stochastic, and
  - volatility of sentiment is stochastic;

  thus, market alternates between periods of quiescence and agitation.

Third Key Feature of Our Model: Market Incompleteness and Frictions

- Typically, general-equilibrium models assume complete financial markets, which simplifies the task of solving for equilibrium.

- However, once regulatory constraints are introduced, financial markets are not complete.

- We identify the equilibrium in the setting with incomplete markets.

Outline

Main Elements of the Model

1. Production technology
   - Stochastic production, with quadratic adjustment costs.
2. Uncertainty and learning by investors
   - Hidden Markov model with "Bayesian" updating
   - The two investors update differently
3. Structure of financial markets
   - Both stocks and bonds can be traded
4. Preferences of each type of investor (there are two types)
   - Additive external habit ("catching up with the Joneses")
5. Financial regulations, which make markets incomplete

Model: Production I

- We assume that there exists a representative firm producing and paying out a single consumption good.
- At each period t the firm uses the capital stock K_t to generate production $Y_t = K_t \times Z_t$, where Z_t denotes the stochastic technology.
- The capital of the firm depreciates at the periodic rate δ, and after investment I_t its law of motion can be described as

$$K_{t+1} = (1 - \delta)K_t + I_t$$

Model: Production II

- We assume that the change in the capital level is subject to quadratic adjustment costs.
- The difference between the production and capital expenses (including adjustment costs) is paid out as dividend D_t:

$$D_t = Y_t - I_t - \frac{\xi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t,$$

with each investor receiving an amount proportional to his stock holdings.

Model: Production III

- Investment I_t is chosen to maximize value of firm $P_{k,t}$ for owner k:

$$P^S_{k,t}(K_t) = \max_{I_t,\ldots,I_{T-1}} \left\{ D_t + E_t \left[\sum_{\tau=t+1}^{T} \frac{M_{k,\tau}}{M_{k,t}} D_\tau \right] \right\}$$

- We assume that the value of the firm is maximized with respect to the expectations of the rational investor.
  - Carceles-Poveda and Coen-Pirani (2007) show that with constant-returns-to-scale production, investors agree on investment decisions even in markets that are not complete.
  - Even though markets are not complete, the pricing kernels of the two investors are similar, and so the investment choices they make are also similar.

Model: Source of Uncertainty

- Uncertainty in the economy is generated by a Hidden Markov Model.
- Economy can be in one of two unobservable productivity states.
- Transition between the unobservable states follows a Markov process.
- While the state of the economy is unobservable for the investors, they observe
  1. productivity realization Z_t, and
  2. a signal
- We assume that productivity and signal can only take on two values.
- So, we have four possible pairs of observations.

Model: Updating of Beliefs

- Investors use the observations to form conditional state probabilities.

- Investor k updates her beliefs about the current state of the economy according to a recursive "forward algorithm", which relies on Bayes, as shown in Baum, Petrie, Soules, and Weiss (1970), Rabiner (1989).

- This forward algorithm is the nonlinear analog for discrete-time discrete-state Markov chains of the Kalman filter, which is applicable to linear stochastic processes.

Model: Heterogeneous Beliefs

- We assume that
  - the realized technology level provides information about the current state of the economy,
  - while the signal is pure noise.

- One investor ("rational") knows signal is pure noise and hence updates her beliefs using the observation probability matrix

$$\Psi^{Rational} = \Psi^{Technology}.$$

- The other investor ("sentiment-prone") believes incorrectly that signal also provides useful information, and uses:

$$\Psi^{Sentiment} = (1 - w) \times \Psi^{Technology} + w \times \Psi^{Signal}.$$

Model: Structure of Financial Markets

- There are two financial assets:
  - a one-period bond (denoted by B_t)
  - a stock (S_t) paying out dividend D_t of the representative firm.

- As the main feature of our model, we will impose a number of exogenous possible regulatory actions such as:
  1. Tobin tax (proportional tax for trading the stock);
  2. Short-sale constraints on the stock; and
  3. Leverage constraints.

Model: Preferences of Investors

- A simple version of "catching up with the Joneses" preferences with additive external habit level:

$$E_k \sum_{t=0}^{T} \beta_k^t \frac{(c_{k,t} - h_k \times C_t)^{1-\gamma_k}}{1-\gamma_k}, \quad \text{where}$$

- h_k is the habit factor;
- $C_t = \frac{1}{K} \sum_{k=1,2} c_{k,t-1}$ is aggregate consumption in previous period
- $\gamma_k > 0$ is equal to relative risk aversion when $h_k = 0$, and is higher than than that otherwise, as $(c_{k,t} - h_k \times C_t)$ becomes smaller.
- E_k denotes the conditional expectation at $t = 0$, under investor k's subjective probability measure;

Investor's Optimization Problem

- **Optimization problem of each (type of) investor**
  - The objective of each investor k is to maximize lifetime utility by choosing consumption, $c(k,t)$, and the portfolio positions in each of the two financial assets,

    subject to the flow budget constraint:

$$\underbrace{c_{k,t} + \theta^S_{k,t} S_{k,t} + \theta^B_{k,t} B_{k,t}}_{\text{uses of funds}} = \underbrace{\theta^S_{k,t-1}\left(S_{k,t} + D_t\right) + \theta^B_{k,t-1}}_{\text{sources of funds}}$$

Effects of Regulatory Measures

1. Tobin tax κ_t affects the individual budget constraint:

   $$c_{k,t} + \theta^S_{k,t} S_{k,t} + \theta^B_{k,t} B_{k,t} + \kappa_t \, S_{k,t} \left| \theta^S_{k,t} - \theta^S_{k,t-1} \right| = \theta^S_{k,t-1} \left(S_{k,t} + D_t \right) + \theta^B_{k,t-1}$$

   Tax revenue is reimbursed to investors as a lump-sum transfer.

2. Short-sale constraint restricts the holdings of the risky asset to be above a predefined limit ρ:

   $$\theta^S_{k,t} \geq \rho, \forall k, t.$$

3. Leverage constraint limits the amount of borrowing, or equivalently, investment in the risky asset, to be less than a specified level α:

   $$\frac{\theta^S_{k,t} \times S_{k,t}}{\theta^B_{k,t} \times B_{k,t} + \theta^S_{k,t} \times S_{k,t}} \leq \alpha, \ \forall k, t,$$

Equilibrium

Equilibrium in this economy is defined as

- consumption policies, $c_{k,t}$, that maximize lifetime expected utility
- portfolio policies, $\theta_{k,t}^{\{B,S\}}$, that finance the optimal portfolio policy
- investment policy, I_t, that maximizes the value of the firm
- price processes for the financial assets, $\{B_t, S_t\}$, such that the following markets clear at each state and date:
  - markets for the stock and bond,
  - market for consumption, and
  - market for investment.

Outline

Calibration of the Model I

- For the quantitative analysis we calibrate our model to match several stylized facts of the U.S. macroeconomy and financial markets.
- We solve model for 30 years, assuming each period in model corresponds to one year, with the last 15 years used as burn-in period.
- All statistics are based on 10,000 simulated paths of economy.
- We use the deprecation rate of 0.08 at an annual frequency.
- We assume the two investors have homogeneous preferences:
  - same risk-aversion, habit parameter, rate of time preference, and initial endowment: 0.50 shares of the firm.
  - Rate of time preference for both investors is 0.9606 p.a.

Calibration of the Model II

- We choose the remaining parameters:
  - risk-aversion
  - habit parameter
  - adjustment costs, and
  - the initial level, volatility, and the growth rate of technology
- to match the following financial and macroeconomic quantities.
  - output volatility
  - investment volatility
  - level of the risk-free interest rate
  - equity risk premium and its volatility.
- Given that data for equity risk premium is for levered firms, we lever the equity premium and its volatility in our model by a factor of 2.

Calibration of Beliefs

- In the Hidden Markov Chain we set the transition probabilities to be 0.95, that is, the hidden states are highly persistent.
- For the initial date assume that it is equally likely that the economy is in either state ($\pi = 0.5$).
- For the baseline calibration we set the level of sentiment of sentiment-prone investor to $w = 0.9$ in the expression below:

$$\Psi^{Sentiment} = (1 - w) \times \Psi^{Technology} + w \times \Psi^{Signal}.$$

Parameter Values

Description	Variable	Value
Preferencs and Beliefs		
Sentiment of irrational Agent	w	0.9
Subject time preference	ρ_k	0.9606
Risk aversion	γ_k	3
Habit parameter	h_k	0.1
Production		
Depreciation	δ	0.08
Volatility of technology	σ_T	4.90%
Technology growth	d_T	0.60%
Adjustment costs	ξ	13

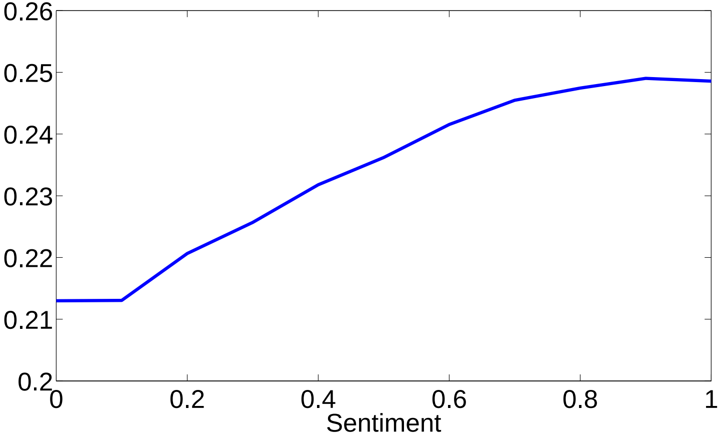
Financial and Business Cycle Statistics: Model vs. U.S. Data

Description	Variable	Model	Data
Macroeconomic variables			
Output volatility	$\sigma(Y)$	3.99%	3.78%
Normalized investment volatility	$\sigma(I)$	2.67%	2.39%
Normalized consumption volatility	$\sigma(C)$	0.93%	0.40%
Correlation between investment & output	$Cor(I, Y)$	0.82	0.96
Correlation between consumption & output	$Cor(C, Y)$	0.95	0.76
Financial variables			
Risk-free rate	r_f	2.30%	1.94%
Interest rate volatility	$\sigma(r_f)$	8.30%	5.44%
Equity premium	$E[R^{ep}]$	3.30%	6.17%
Equity premium volatility	$\sigma(R^{ep})$	21.70%	19.40%
Sharpe ratio	$E[R^{ep}]/\sigma(R^{ep})$	15%	32%

Effect of Sentiment on Financial Variables

Sentiment is measured by weight put on uninformative signal by “sentiment-prone” investor.

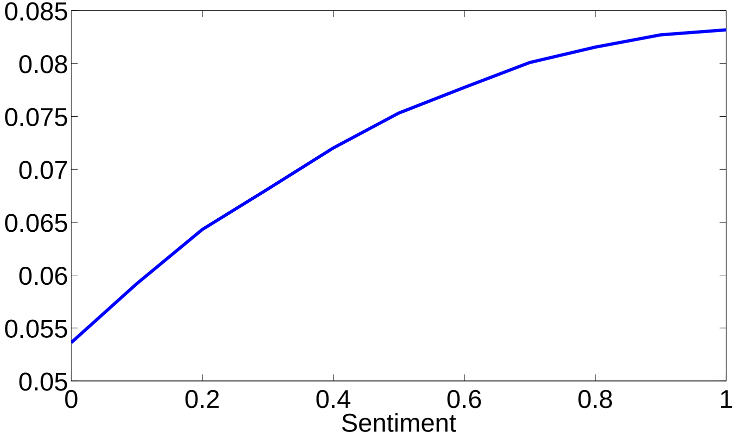
Panel A: Volatility of Stock Returns



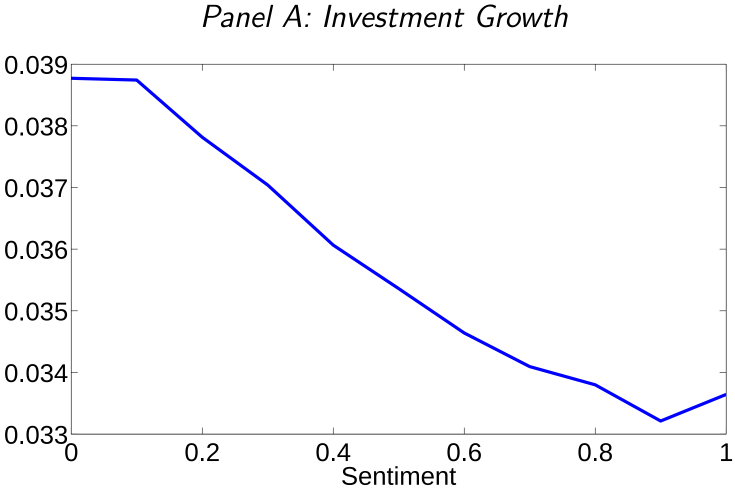
Effect of Sentiment on Financial Variables

Sentiment is measured by weight put on uninformative signal by "sentiment-prone" investor.

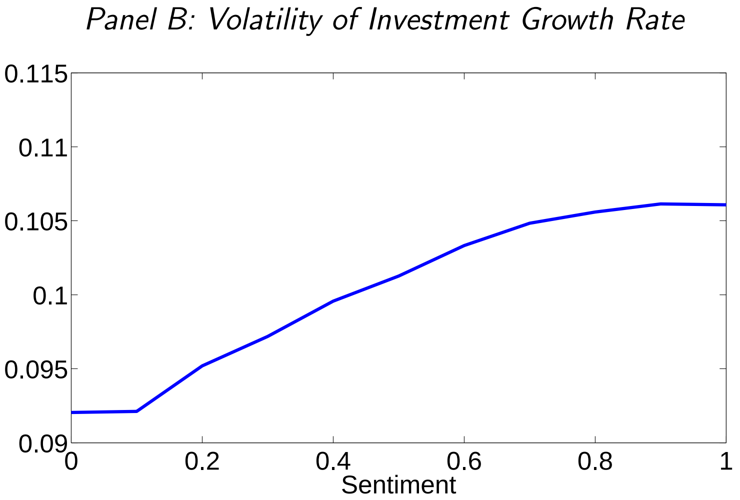
Panel B: Volatility of the Interest Rate



Effect of Sentiment on Investment Growth



Effect of Sentiment on Investment Growth



Outline

1. Motivation, Objective, and Contribution
2. The Model
3. Analysis of Measures to Regulate Stock-Return Volatility
4. Conclusion

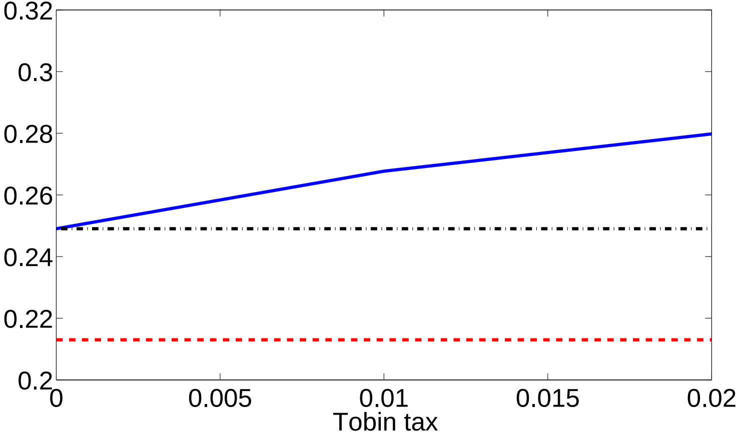
Effect of Regulatory Measures

- We illustrate the effects of regulatory measures using figures.

- Each plot has **three lines**:
  - The **red line** depicts case when *both* investors learn rationally;
  - The **black line** depicts case of excessive volatility due to "sentiment-prone" trading but without regulations;
  - The **blue line** depicts case with a particular regulatory measure in the economy with excessive volatility.

Volatility of Stock Returns

Red: Both rational; Black: One sentiment-prone, no regulation; Blue: With regulation



Volatility of Stock Returns

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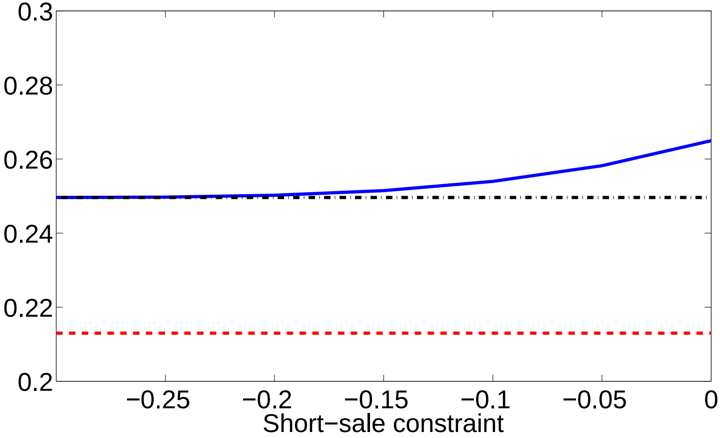

Volatility of Stock Returns

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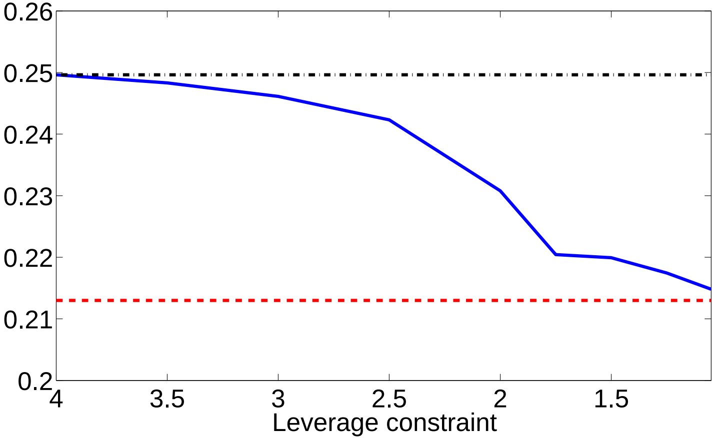
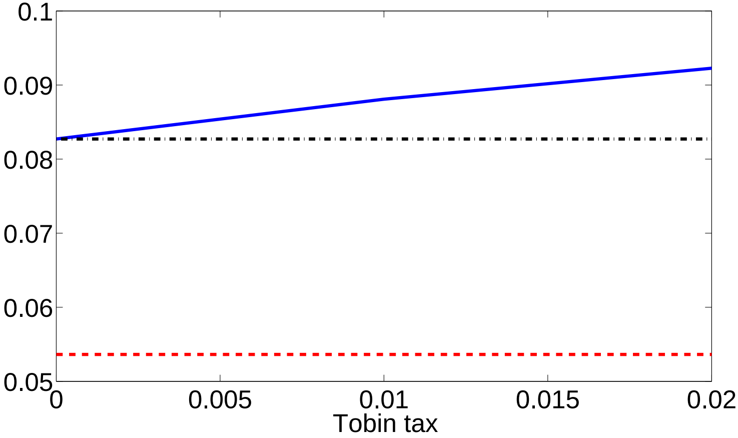


Volatility of the Risk-free Interest Rate

Red: Both rational; Black: One sentiment-prone, no regulation; Blue: With regulation



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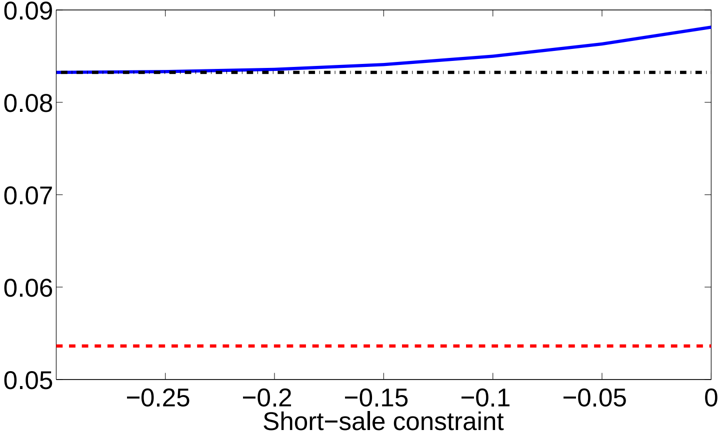

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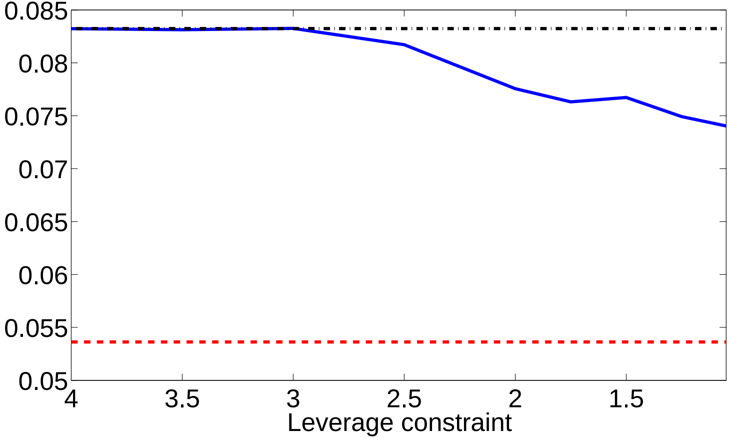
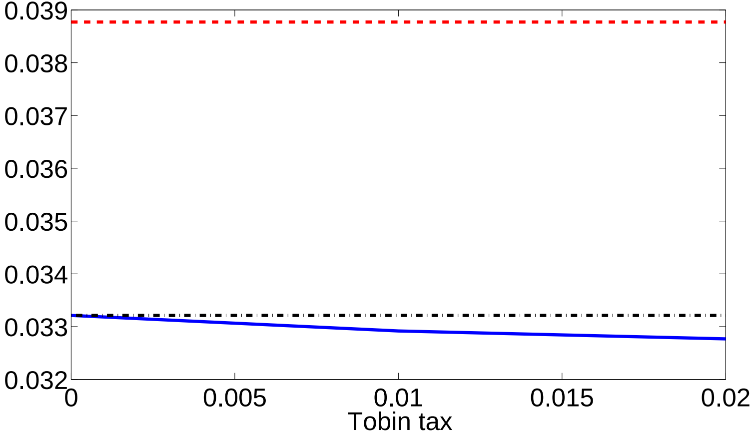


Investment Growth

Red: Both rational; Black: One sentiment-prone, no regulation; Blue: With regulation



Investment Growth

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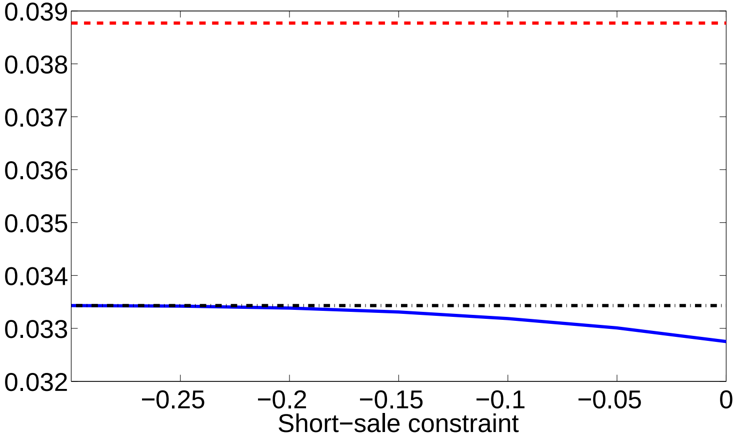

Investment Growth

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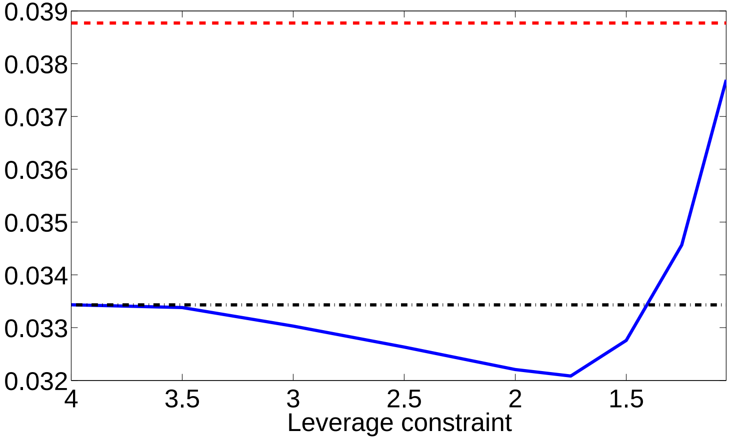
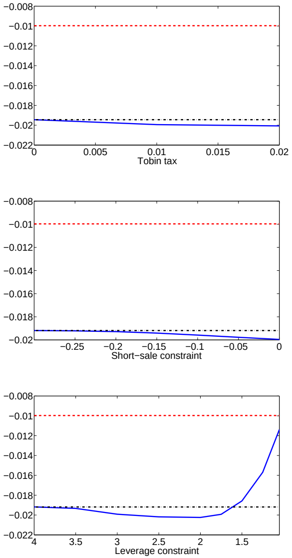
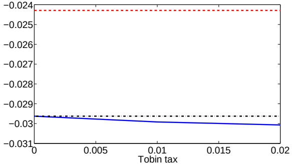
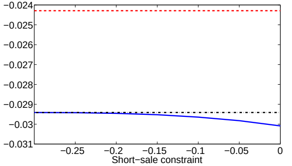


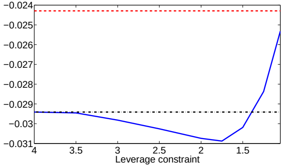
Consumption Growth



Output Growth







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Conclusion

- We quantitatively assess regulatory measures such as
  - Tobin financial transaction tax
  - Borrowing constraints
  - Shortsale constraints
- This analysis allows one to compare and understand how these measures influence
  - stock market level and volatility
  - output growth rate and volatility
  - level of interest rate and its term structure
  - trading volume and liquidity in financial markets

Summary of Findings

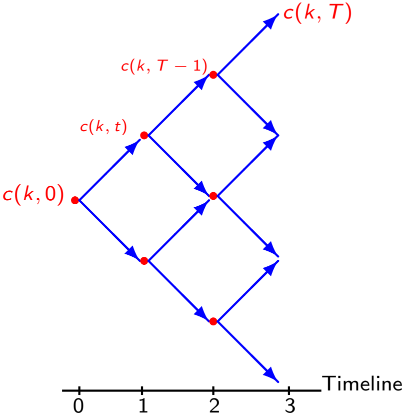
Code:

- Blue is positive effect
- Red is negative effect

Quantity	**Tobin Tax**	**Shortsale Constraint**	**Leverage Constraint**
Financial Markets			
Volatility	Higher	Higher	Lower
Production			
Investment and output	Reduced	Reduced	Increased
Volatility	Increased	Increased	Mixed
Consumption			
Growth	Lower	Lower	Much higher
Volatility	Higher	Higher	Lower

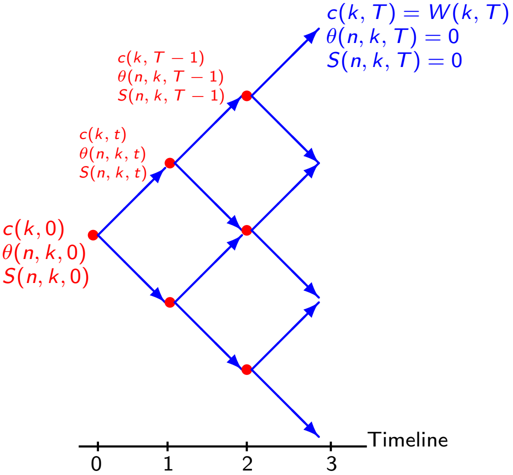
Appendix: Method for Solving the Model If Markets Were Complete . . .

- Solve at **each** node for optimal consumption by equating marginal utility across agents; **then**, get prices, then portfolios.



If Markets are Incomplete: Global Approach

- Solve **simultaneously** for all unknowns at all dates and states.



Problem in Implementing the Global Approach

- Two problems in implementing the global approach:

1. Global problem is path dependent; thus, large number of equations, which grow quickly with number of time periods.

2. TC give a rise to the no-trade region:
   - Inside of the no-trade region agents disagree on the asset prices;
   - System of equations to be solved changes if some assets are not traded

Recursive Approach to Solving the Model

- Two problems when solving for equilibrium recursively

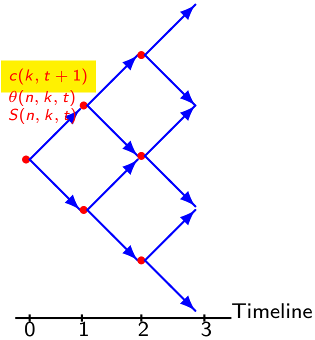
1. The system is not backward-only, but is backward-forward
   - This is because prices are determined going forward (current prices depend on future consumption)

$$\text{Stock Price}_t = E_t^k \left[\left(\frac{\text{Marginal Utility of } c_{t+1}}{\text{Marginal Utility of } c_t} \right) \times \left(\text{Total payoff}_{t+1} \right) \right]$$

2. No-trade region leads to the change of optimality conditions; this is a problem with "occasionally binding constraints."

Solution to First Problem for Recursive Solution

- **Solution**: Do time shift proposed in Dumas and Lyasoff (2012)
- Solve recursively at each node for
  - current portfolio: $\theta(n, k, t)$ and current prices: $S(n, k, t)$
  - future consumption: $c(k, t+1)$.



Recursive Approach: After the Time Shift

- Solve recursively at each node for
  - current portfolio: $\theta(n, k, t)$
  - current prices: $S(n, k, t)$
  - future consumption: $c(k, t+1)$.
- Use as state variables $\theta_{k,t-1}$ and $c_{k,t}$
- System is now backward only, and also path-independent
- At time 0, solve one forward step to satisfy the initial conditions.

Solution to Second Problem for Recursive Solution

- **Solution**: Exploit structure of proportional transactions costs
  - Proportional TC: $|\theta(t) - \theta(t-1)| \times S(t) \times \kappa$:
  - Solve as a function of trading decision: {sell, no-trade, buy}
  - Derivative can take only the values $\{-1, 0, 1\} \times S \times \kappa$
  - As a result get the no-trade-region bounds
  - Inside of no-trade region no trade occurs, i.e., $\theta(n, k, t){=}\theta(n, k, t-1)$
  - Solve reduced system inside no-trade region, knowing no trade occurs

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