Why Does Equal-Weighting Outperform Value- and Price-Weighting?

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Ancient Portfolio Theory

- Rabbi Issac bar Aha (Talmud, 4th Century): **Equal allocation**
  A third in land, a third in merchandise, a third in cash.

- We call this the $1/N$ rule: equal allocation in the N available assets.

- It is also called "**naive diversification**" because it ignores that
  - assets may have different expected returns;
  - assets may be correlated.

- Recently, it has been called: **Balanced indexing**.

Modern Portfolio Theory: Strengths and Weaknesses

- **Harry Markowitz (1952)**
  - Need to take into account the correlation between assets.
- **James Tobin (1958)**
  - Two-fund separation.
- **William Sharpe (1964)**
  - The two funds are the risk-free asset and the value-weighted market portfolio, and only systematic risk matters for returns.
- **Paul Samuelson (1969)**
  - If the investment environment is not changing, then the single-period portfolio is optimal even in a multiperiod setting.
- **Robert Merton (1971)**
  - If the investment environment is changing, then the optimal portfolio is the single period portfolio plus some "hedge" portfolios.

Two things to note . . .

- **Harry Markowitz** Nobel Prize, 1990
- **James Tobin** Nobel Prize, 1981
- **William Sharpe** Nobel Prize, 1990
- **Paul Samuelson** Nobel Prize, 1970
- **Robert Merton** Nobel Prize, 1990

1. Modern portfolio theory is very elegant.
2. Actual performance of optimal portfolios is very poor.
   - Michaud (1989) calls them "**error-maximizing**" portfolios

Recent Models: Evaluating Out-of-Sample Performance

- **DeMiguel, Garlappi, and Uppal (2009)** evaluate out-of-sample performance of recent portfolio models.

- **Compare** out-of-sample performance of $1/N$ to **14 model** across **7 datasets**.

- **Find** that **none** of the optimal portfolio-selection models consistently dominates $1/N$.

List of Portfolio-Selection Models Evaluated

Naive

0 $1/N$ with rebalancing *(benchmark strategy)*

Classical approach that ignores estimation error

1 Sample-based mean-variance

Bayesian approach to estimation error

2 Bayesian diffuse-prior
3 Bayes-Stein
4 Bayesian Data-and-Model

Moment restrictions

5 Minimum-variance
6 Value-weighted market portfolio
7 MacKinlay and Pástor (2000) missing-factor model

Portfolio constraints

8 Classic mean-variance with shortsale constraints
9 Bayes-Stein with shortsale constraints
10 Minimum-variance with shortsale constraints
11 Minimum-variance with generalized constraints

Optimal combinations of portfolios

12 Kan and Zhou (2007) "three-fund"
13 Mixture of minimum-variance and $1/N$
14 Garlappi, Uppal, and Wang (2007) multi-prior robust portfolio

List of Datasets Considered

1	**Ten sector** portfolios of the S&P500 and MKT Source: Roberto Wessels	10+1	1981–2002
2	**Ten industry** portfolios and MKT Source: Ken French's website	10+1	1963–2004
3	**Eight country indexes** and World Mkt Source: MSCI	8+1	1970–2001
4	**SMB and HML** portfolios and **MKT** Source: Ken French's website	2+1	1963–2004
5	**Twenty Size & Book-to-Market** portfolios, **MKT** Source: Ken French's website	20+1	1963–2004
6	**Twenty Size and Book-to-Market** portfolios and the **MKT, SMB, and HML** portfolios Source: Ken French's website	20+3	1963–2004
7	**Twenty Size and Book-to-Market** portfolios and the **MKT, SMB, HML & UMD** portfolios Source: Ken French's website	20+4	1963–2004
8	**Simulated data** Source: Market model	$\{10, 25, 50\}$	2,000 years

What Other Researchers Find

- **Jacobs, Muller, and Weber (2010)**
  - Extend data period: 1973 to 2009
  - Extend analysis across countries, and take perspective of Euro investor
  - Extend analysis across asset classes (stocks, bonds, and commodities)
  - Extend list of models of optimal portfolio selection studied
  - Extend performance evaluation metrics
  - Report that:

    Analyzing more than 5,000 heuristics, our results show that in fact almost any form of **well-balanced allocation** *over asset classes offers similar diversification gains as even recently developed portfolio optimization approaches.*

- **Conclusion:** Estimation error leads to poor performance of "optimal" models, relative to equal-weighted portfolio.

Main Question

- **Question:**
  How does equal weighting compare to value- and price-weightings?

- Important to understand difference in performance of equal- and value-weighted portfolios because of the value-weighted portfolio's
  - central role in asset pricing (CAPM);
  - importance as a benchmark against which managers are evaluated;
  - growing popularity as an investment strategy.

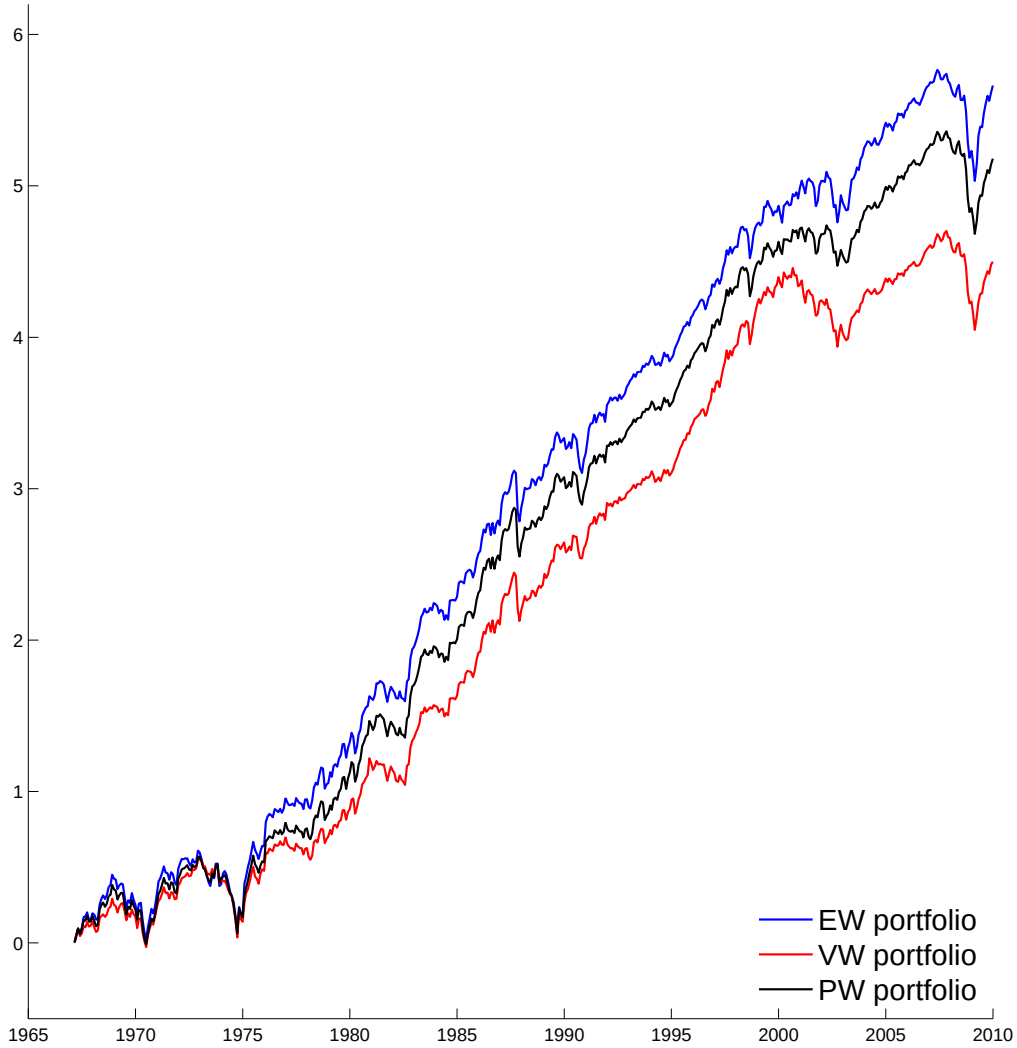
Our Objective

1. Compare the performance of the three most popular weighting rules:
   - equal-weighted (EW),
   - value-weighted (VW), and
   - price-weighted (PW) portfolios.

2. Understand reasons for differences in performance

3. Relate difference in performance to
   - stock characteristics;
   - factor exposure of portfolios;
   - trading required to rebalance the portfolios.

A Picture of Cumulative Portfolio Returns



Main Findings

1. **Overall performance:** Equal-weighted portfolio has
   - higher total return, alpha, Sharpe ratio, CEQ, & less negative skewness
   - **but** higher volatility and kurtosis.

2. **Systematic return:** Equal-weighted portfolio has
   - higher systematic return than value-weighted portfolio
   - because of higher exposure to market, size, and value risk factors

3. **Unsystematic return (alpha):** Equal-weighted portfolio has
   - higher alpha, arising from rebalancing,
   - which makes equal-weighting a contrarian strategy,
   - which exploits reversal in prices at monthly frequency.

Data is from CRSP and COMPUSTAT

- Monthly data for constituents of S&P500 index from February 1967 to December 2009: 1449 stocks.
- Construct equal-, value-, and price-weighted portfolios from 100 randomly selected stocks from constituents of S&P500 index.
- To ensure that our results are not driven by the choice of stocks that we select, we resample 1000 times the stocks for the portfolios.
- If S&P announces decision to remove a particular stock in our portfolio, we also remove this stock from our portfolio and randomly choose another stock to replace it.
- The company characteristics used in our analysis, are constructed using the monthly and daily CRSP and COMPUSTAT databases.

Performance Metrics: Return and Risk

- **Return measures:**
  - Total return
  - Systematic return
  - Unsystematic return (alpha) from one- and four-factor model
- **Risk measures:**
  - Volatility
  - Skewness
  - Kurtosis
  - Maximum drawdown = max % loss of portfolio value over last 12 mo.

Performance Metrics: Return and Risk Tradeoff

- **Risk-return tradeoff measures:**
  - Sharpe ratio
  - Certainty equivalent return
    of myopic CRRA investor with relative risk aversion of 2 & 5

- **Transactions costs:**
  - We compute the transaction costs for rebalancing the portfolios assuming 50 b.p. for buying or selling the stock.

Returns: Total, Systematic, and Alpha

Metrics (per year)	Performance before transaction costs			Performance net of transaction costs		
	EW	VW	PW	EW	VW	PW
Total Return	**0.1319** (1.00)	**0.1048** (0.00)	**0.1207** (0.00)	**0.1283** (1.00)	**0.1042** (0.00)	**0.1195** (0.00)
Systematic Return	**0.1144** (1.00)	**0.0988** (0.00)	**0.1140** (0.80)	**0.1144** (1.00)	**0.0988** (0.00)	**0.1140** (0.39)
Four-factor alpha	**0.0175** (1.00)	**0.0060** (0.04)	**0.0067** (0.00)	**0.0139** (1.00)	**0.0054** (0.07)	**0.0056** (0.00)

	EW−VW	EW−PW	EW−VW	EW−PW
Diff in Total Return	271 bp	112 bp	241 bp	88 bp
Diff in Systematic Return	156 bp	4 bp	156 bp	4 bp
Diff in Alpha	115 bp	108 bp	85 bp	83 bp

Returns and Transactions Costs

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Turnover	0.7216 (1.00)	0.1231 (0.00)	0.2296 (0.00)	0.7216 (1.00)	0.1231 (0.00)	0.2296 (0.00)
Transaction costs	0.0036 (1.00)	0.0006 (0.00)	0.0011 (0.00)	0.0036 (1.00)	0.0006 (0.00)	0.0011 (0.00)

Risk

Metrics (per year)	Performance before transaction costs			Performance net of transaction costs		
	EW	VW	PW	EW	VW	PW
Volatility	**0.1790** (1.00)	**0.1583** (0.00)	**0.1646** (0.00)	**0.1790** (1.00)	**0.1583** (0.00)	**0.1646** (0.00)
Skewness	**-0.3266** (1.00)	**-0.3860** (0.43)	**-0.4996** (0.00)	**-0.3287** (1.00)	**-0.3862** (0.22)	**-0.4998** (0.00)
Kurtosis	**5.5305** (1.00)	**4.8372** (0.01)	**5.3608** (0.39)	5.5321 (1.00)	4.8380 (0.00)	5.3614 (0.19)
Max Drawdown	**0.1154** (1.00)	**0.1047** (0.04)	**0.1078** (0.00)	**0.1164** (1.00)	**0.1049** (0.01)	**0.1081** (0.00)

Return-Risk-Tradeoff

Metrics (per year)	Performance before transaction costs			Performance net of transaction costs		
	EW	VW	PW	EW	VW	PW
Sharpe Ratio	**0.4275** (1.00)	**0.3126** (0.00)	**0.3966** (0.05)	**0.4074** (1.00)	**0.3088** (0.00)	**0.3896** (0.13)
CEQ, $\gamma = 2$	**0.0994** (1.00)	**0.0793** (0.00)	**0.0930** (0.01)	**0.0957** (1.00)	**0.0787** (0.00)	**0.0918** (0.07)

Inferences Based on Performance Measures

- Equal-weighted portfolio has the highest total return.
- Equal-weighted portfolio has highest volatility.
- Thus, the good Sharpe ratio of equal-weighting is a consequence entirely of its return.
- In the rest of the presentation, we focus on explaining differences in returns across the three weighting schemes.

Sources of the Differences in Returns

Question:

- What can explain the difference in returns across weighting methods?
- Three possible sources driving differences in returns:

1. Stock characteristics
2. Factor exposure
3. Rebalancing strategy

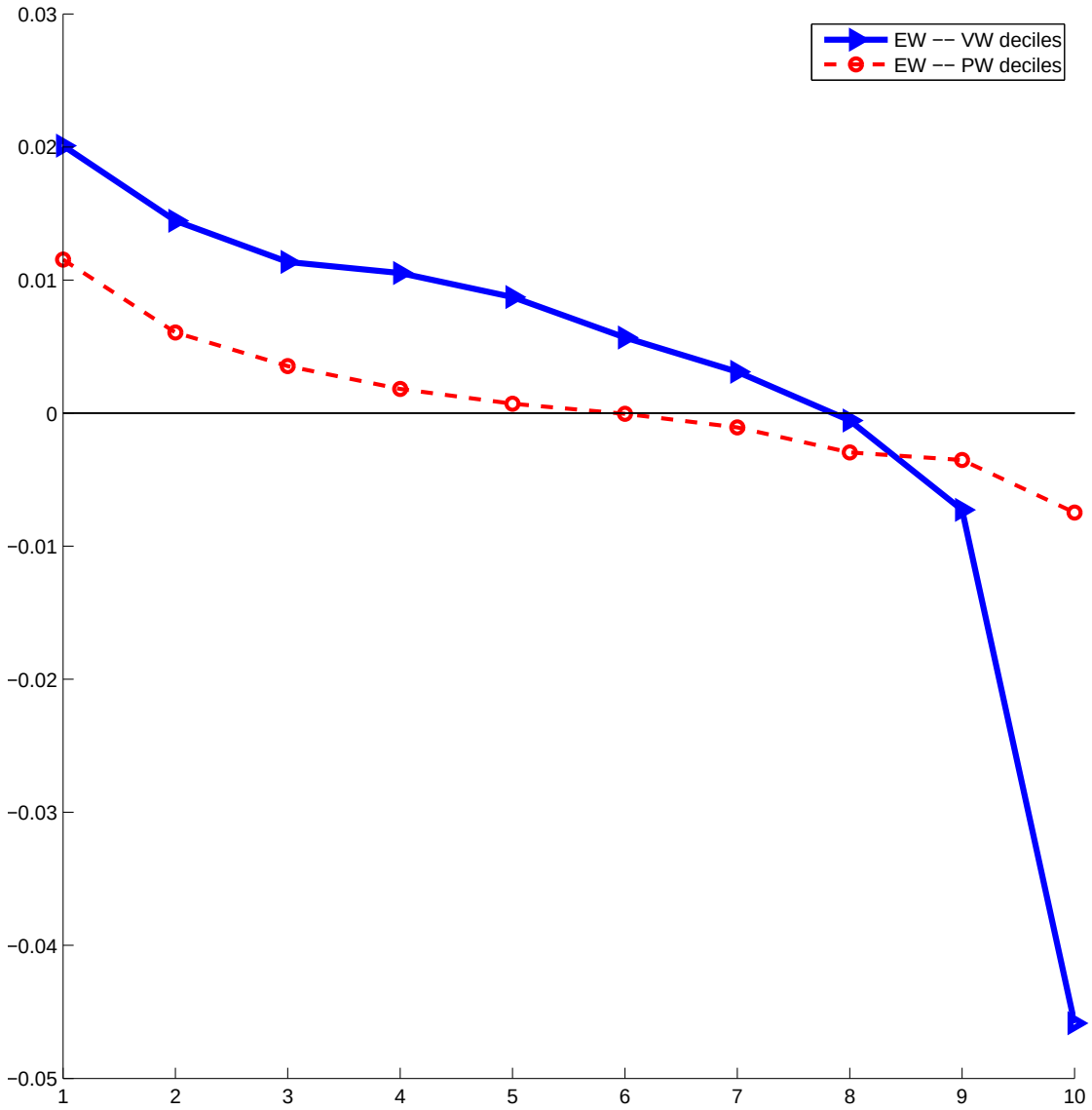
Total Portfolio Returns and Stock Characteristics

1. We study eight characteristics of stocks:
   size, book-to-market, 3- and 12-month momentum, reversal, price, liquidity, and idiosyncratic volatility.

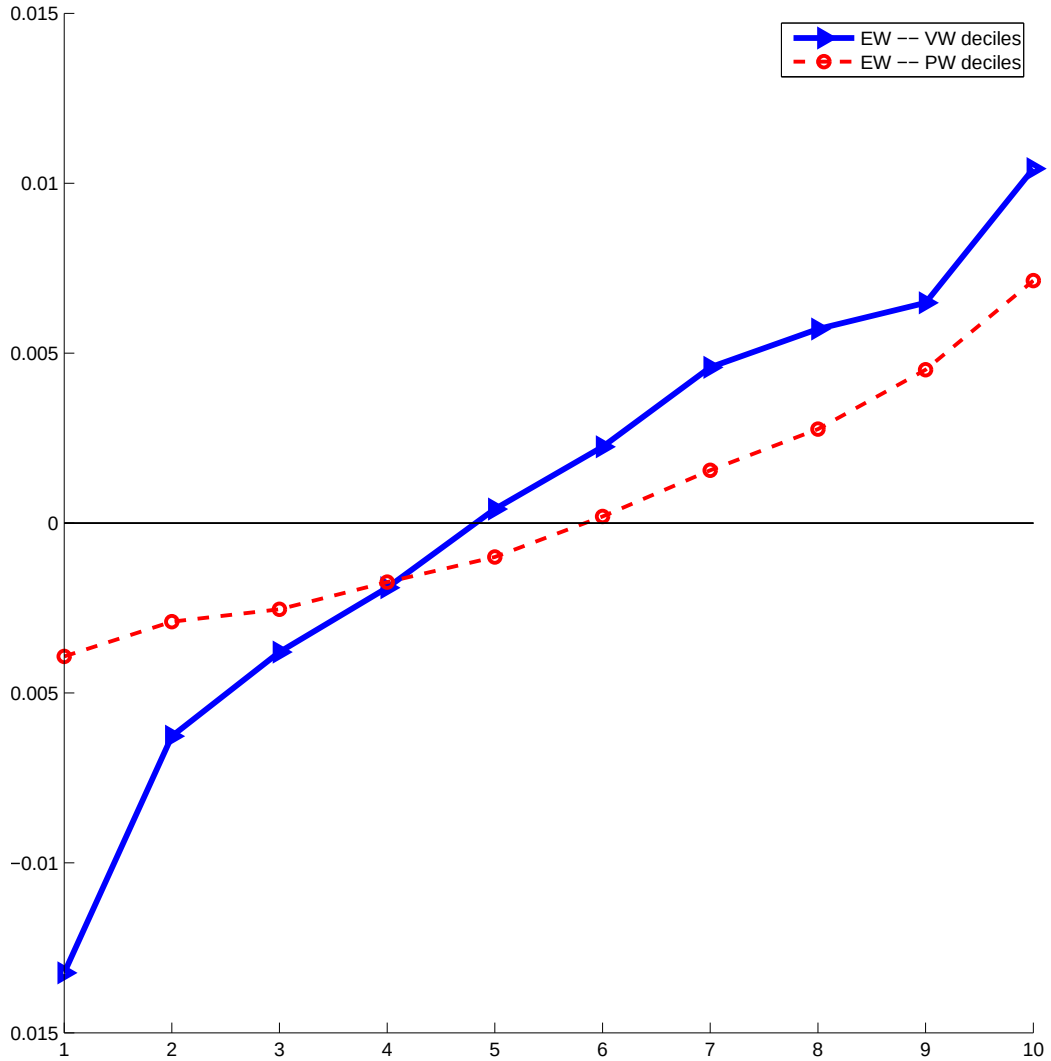
2. Construct portfolio deciles from the previously resampled 1000 portfolios and sort them by the selected characteristic.

3. Use non-parametric test of Patton and Timmermann (2010) to test
   - if there is a **monotonic relation** between stock characteristic, and
   - differences in total returns; that is, EW – VW and EW – PW:
     - EW − VW = 271 bp p.a.
     - EW − PW = 112 bp p.a.

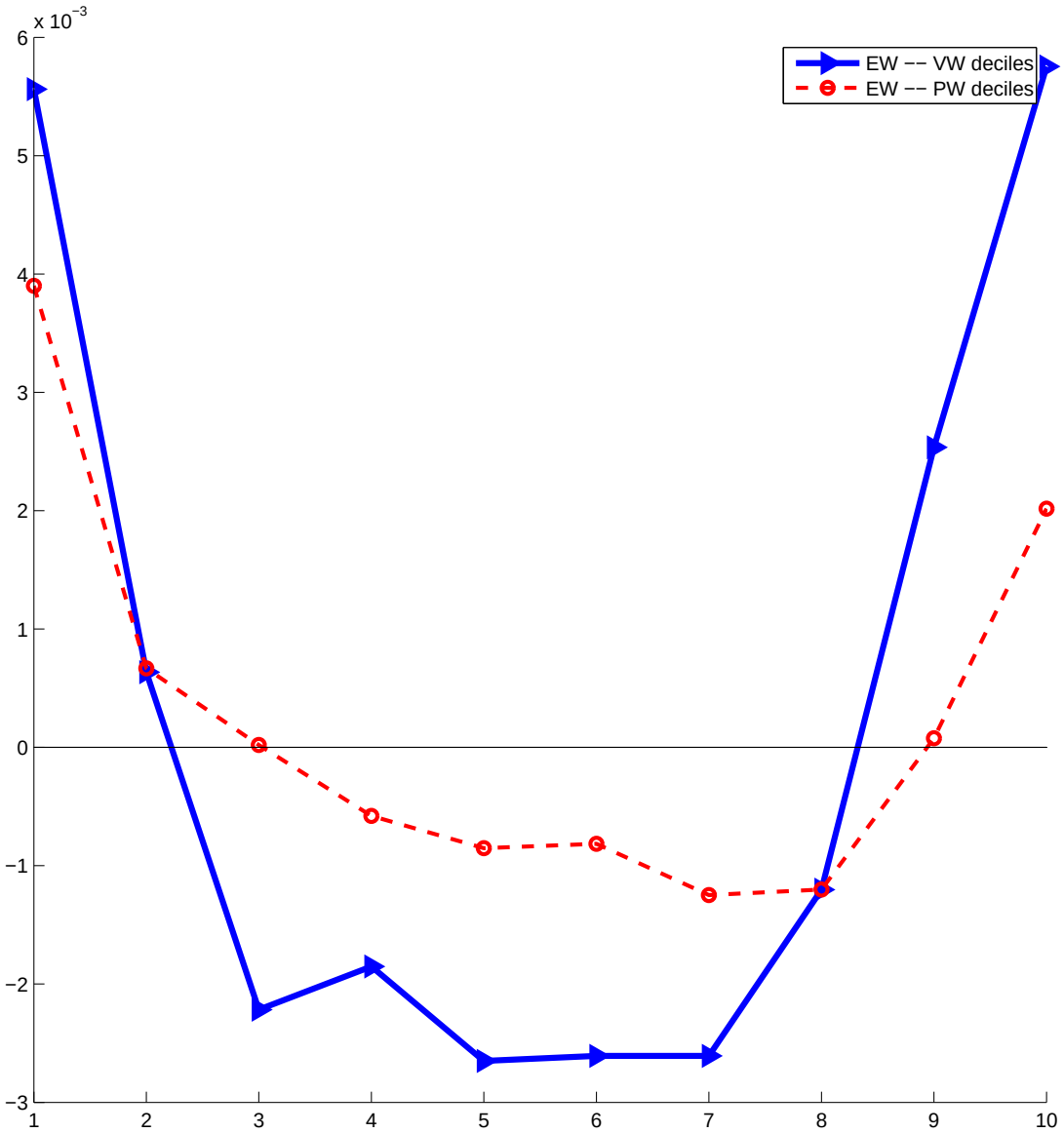
Size and Differences in Portfolio Returns



Idiosyncratic Volatility and Differences in Portfolio Returns



Reversal and Differences in Portfolio Returns



Conclusion: Differences in Returns and Characteristics

- We have used a nonparametric approach to identify which characteristics of stocks could be driving differences in total return:

- EW – VW has a monotonically
  - decreasing relation with size, price, and liquidity;
  - increasing relation with idiosyncratic volatility and book-to-market.

- EW – PW has a monotonically
  - decreasing relation with size, price, liquidity, and 12-month momentum;
  - increasing relation with idiosyncratic volatility.

Understanding Differences in Systematic Returns

- We use the standard four-factor model of Fama and French
- To decompose the total return into a
  - systematic return, which is compensation for bearing factor risk, and
  - alpha, which is the unsystematic component.

Systematic Return Implied by Four-Factor Model

Portfolio	α_4	β_{mkt}	β_{smb}	β_{hml}	β_{umd}	R^2	MSE
EW	**0.0175**	**1.0797**	**0.0955**	**0.3027**	**-0.1379**	0.9361	0.0002
VW	**0.0060**	**0.9890**	**-0.2024**	**0.0234**	**-0.0130**	0.9330	0.0001
PW	**0.0067**	**1.0311**	**-0.0249**	**0.1790**	**-0.0063**	0.9351	0.0001

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VW	**0.0060**	**0.9890**	**-0.2024**	**0.0234**	**-0.0130**	0.9330	0.0001
PW	**0.0067**	**1.0311**	**-0.0249**	**0.1790**	**-0.0063**	0.9351	0.0001

- **EW − VW total = 271 bp**
  - 175 − 60 = 115 bp or 42% comes from the difference in alpha;
  - **271** − 115 = 156 bp or 58% from the difference in systematic return.
- **EW − PW total = 112 bp**
  - 175 − 67 = 108 bp or 96% comes from the difference in alpha;
  - **112** − 108 = 4 bp or 4% from the difference in systematic return.

Understanding Differences in Alpha

- We demonstrate that:
  - source of the extra alpha of the EW portfolio arises from
  - contrarian rebalancing each month required to maintain equal weights
  - which exploits the reversal in stock prices;
  - see, Jegadeesh (1990) and Jegadeesh and Titman (1993, 2002).

Perform Two Experiments in Opposite Directions

1. Reduce rebalancing frequency of the equal-weighted portfolio from 1 month to 6 months, to 12 months.
   - Question: Does alpha decrease?
   - Answer: Yes.
     - Alpha decreases from 175 bp, to 115 bp, to 65 bp.

2. Fix weights of VW and PW for 6 months and 12 months.
   - Question: Does alpha increase?
   - Answer: Yes.
     - for VW: Alpha changes from 60 bp, to 125 bp, to 148 bp;
     - for PW: Alpha changes from 67 bp, to 112 bp, to to 145 bp.

Alpha as Rebalancing Frequency of Equal-Weighted Portfolio is Decreased

Metrics (per year)	Rebalancing frequency				
	Base case: 1 month			6 month	12 month
	EW	VW	PW	EW	EW
Four-factor alpha	0.0175	0.0060	0.0067	0.0115	0.0065
	(1.00)	(0.04)	(0.00)	(0.00)	(0.00)

Alpha When Weights of Value- and Price-Weighted Portfolios Held Fixed

Metrics (per year)	Weights fixed for						
	Base case: 1 month			6 month		12 month	
	EW	VW	PW	VW	PW	VW	PW
Four-factor alpha	0.0175	0.0060	0.0067	0.0125	0.0112	0.0148	0.0145
	(1.00)	(0.04)	(0.00)	(0.39)	(0.04)	(0.65)	(0.30)

Robustness Tests

1. Different samples of stocks
   - stocks constituents of S&P400: July 1991 to Dec 2009 — 1162 stocks
   - stocks constituents of S&P600: Nov 1994 to Dec 2009 — 1681 stocks
2. Different number of stocks in the portfolio
   - 30, 50, 100, 200 and 300 stocks in the portfolio
3. Different rebalancing frequency to understand alpha
   - from monthly to 5 years
4. Different economic conditions when initiated investment
   - Peak of March 2001;
   - Trough of November 2001;
   - Peak of December 2007.

Portfolio Performance for Different Start Dates Over the Business Cycle

Metrics (per year)	Start of the investment period								
	Peak of March 2001			Trough of November 2001			Peak of December 2007		
	EW	VW	PW	EW	VW	PW	EW	VW	PW
Total Return	0.0754	0.0236	0.0566	0.0931	0.0417	0.0746	-0.0143	-0.0788	-0.0674
	(1.00)	(0.00)	(0.00)	(1.00)	(0.00)	(0.00)	(1.00)	(0.00)	(0.00)
Systematic Return	0.0318	0.0288	0.0304	0.0419	0.0366	0.0354	-0.0747	0.0178	-0.0291
	(1.00)	(0.22)	(0.24)	(1.00)	(0.15)	(0.02)	(1.00)	(0.00)	(0.01)
Four-factor alpha	0.0436	-0.0052	0.0262	0.0512	0.0052	0.0392	0.0604	-0.0965	-0.0382
	(1.00)	(0.00)	(0.00)	(1.00)	(0.00)	(0.05)	(1.00)	(0.00)	(0.00)
Volatility	0.1977	0.1619	0.1675	0.1967	0.1598	0.1661	0.2985	0.2322	0.2558
	(1.00)	(0.00)	(0.00)	(1.00)	(0.00)	(0.00)	(1.00)	(0.00)	(0.00)
Skewness	-0.4788	-0.6941	-0.9326	-0.4943	-0.8028	-1.0137	-0.2545	-0.5324	-0.6001
	(1.00)	(0.07)	(0.00)	(1.00)	(0.03)	(0.00)	(1.00)	(0.03)	(0.00)
Max Drawdown	0.1445	0.1303	0.1285	0.1391	0.1244	0.1243	0.2631	0.2365	0.2502
	(1.00)	(0.08)	(0.00)	(1.00)	(0.07)	(0.00)	(1.00)	(0.13)	(0.10)

Portfolio Performance for Different Start Dates Over the Business Cycle

Metrics (per year)	Start of the investment period								
	Peak of March 2001			Trough of November 2001			Peak of December 2007		
	EW	VW	PW	EW	VW	PW	EW	VW	PW
Turnover	0.7615	0.1280	0.1700	0.7424	0.1266	0.1694	0.9638	0.1414	0.1516
	(1.00)	(0.00)	(0.00)	(1.00)	(0.00)	(0.00)	(1.00)	(0.00)	(0.00)
Transaction costs	0.0038	0.0006	0.0009	0.0037	0.0006	0.0008	0.0048	0.0007	0.0008
	(1.00)	(0.00)	(0.00)	(1.00)	(0.00)	(0.00)	(1.00)	(0.00)	(0.00)
Sharpe Ratio	0.2639	0.0037	0.1993	0.3615	0.1252	0.3163	-0.0795	-0.3780	-0.2995
	(1.00)	(0.00)	(0.03)	(1.00)	(0.00)	(0.09)	(1.00)	(0.00)	(0.00)
CEQ, $\gamma = 2$	0.0353	-0.0035	0.0274	0.0535	0.0153	0.0457	-0.1033	-0.1336	-0.1343
	(1.00)	(0.00)	(0.09)	(1.00)	(0.00)	(0.09)	(1.00)	(0.14)	(0.02)

Conclusion and Summary of Main Results

1. Equally-weighting outperforms value- and price-weightings.
   - Source of outperformance is higher return rather than lower risk.
2. Higher total returns of equal-weighted portfolio is
   - monotonically related to size, price, liquidity and idiosyncratic volatility.
3. Higher systematic returns of equal-weighted portfolio arises from its
   - higher exposure to market, size, and value factors.
4. Higher alpha of the equal-weighted portfolio arises from the
   - monthly rebalancing required to maintain equal weights, and
   - does not depend on the choice of initial weights

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